6.3. Relations: Formal Semantics

When providing a semantic interpretation for predicate letters in the previous chapter, we began with a domain of discourse populated by one or more objects, and then provided each predicate letter in the model with a (possibly empty) set of those objects to 'apply' to.

$\mathbb{D}$: {**NEKO**, **LUCRETIA**, **JACK**}

a: **NEKO** **b**: **JACK** **c**: **LUCRETIA**

$\mathbf{G^1}$: {**LUCRETIA**} $\mathbf{I^1}$: {**NEKO**, **LUCRETIA**}
$\mathbf{H^1}$: {**NEKO**, **JACK**} $\mathbf{J^1}$: { }

So if "G^1" stands for "is human", "H^1" for "is a cat", "I^1" for "is female" and "J^1" for "is a unicorn", then this model is a situation where Lucretia's human, Lucretia and Neko are female, Neko and Jack are cats, and nobody's a unicorn. We could think of the extension of a predicate as all the **'cases'** of the feature discussed: all the examples (in this situation) of human-ness, cat-hood, female-ness, and unicornicity.

That suggests a natural adaptation of the semantics to relations: the extension of a relation will likewise be just the cases of that relation appearing in a given situation. For example, the extension of "likes" will be just the cases of liking occurring in that context. If in a certain situation Neko likes Jack and Jack likes Lucretia, there are two cases of liking going: one involving Neko and Jack, another involving Jack and Lucretia.

We might consider adding that to the model as follows (translating "likes" as "K^2").

☠ Extension of "K^2"? ☠

G^2ab: **a likes b**

G^2: {{**NEKO**, **JACK**}, {**JACK**, **LUCRETIA**}}

But keep in mind a point made earlier: it may be true in a certain situation that "Neko likes Jack" but false in that situation that "Jack likes Neko". Hence those two sentences report **different cases** of liking, which can't be slopped together in the semantics – on pain of confusing true sentences with false ones.[1]

To stress the importance of who's doing what in a certain case, we use **ordered pairs** in the semantics to depict a particular case of a relation – marked as ordered by using angle brackets "<" and ">". So just as "Neko likes Jack" and "Jack likes Neko" make different claims, the two ordered pairs in the following extension are **different** cases of liking – the first where Neko likes Jack, the second where Jack likes Neko.

G^2: {**< NEKO**, **JACK >**, **< JACK**, **NEKO >**}

Likewise with a three-place relation: it's one thing for Trixie to give Old Reliable to Trixie, and something else for Trixie to give Old Reliable to Kitty. In the following model, with only one **ordered triple** in the extension of "K^3," Kitty gave Old Reliable to Trixie and nobody gave anything else to anybody.

K^3abc: **a gave b to c**

$\mathbb{D}$: {**KITTY**, **OLD RELIABLE**, **TRIXIE**}

d: **KITTY** e: **OLD RELIABLE** f: **TRIXIE**

K^3: {**< KITTY**, **OLD RELIABLE**, **TRIXIE >**}

[1] The technical worry here turns on the fact that sets with exactly the same members are counted mathematically as one and the same set. For example, the sets {**Neko**, **Jack**} and {**Jack**, **Neko**} count in set theory as the same set, just depicted in two different ways. By contrast, **Neko's liking Jack** isn't the same as **Jack's liking Neko**. So just listing the actors in a set leaves out the important matter of who's the 'doer' and who's the 'do-ee'.

In general: the semantic value of an *n*-place relation letter in a particular model is the **extension of** that ***n*-place relation letter**: a (possibly empty) **set of ordered *n*-tuples of objects** from the domain of that model (so: order pairs for a 2-place relation letter, ordered triples for a 3-place relation letter, and so on).[2]

> **The semantic value of an *n*-place relation letter in model *M* is a (possibly empty) set of ordered *n*-tuples of objects drawn from the domain of *M*.**

So the sentence "G^2ab" is true in a certain model if and only if the extension of "G^2" includes an ordered pair of objects where the first member of the ordered pair is the object named "a," and the second member of the ordered pair is the object named "**b**". And sentence "G^2ab" is false in a model if that ordered pair doesn't appear in the extension of "G^2ab" in that model.

An ordered n-tuple of objects can have the same object show up more than once. For instance, the following is an ordered pair where Neko holds both positions.

< NEKO, NEKO >

And that's a good thing, for in a situation where Neko likes herself, Neko is both 'liker' and 'like-ee'. Hence a model where the sentence "Neko likes herself" (or formal translation "G^2aa") is true is a model where the above ordered pair shows up in the extension of "likes" (or of the formal translation of "likes" – say, "G^2").

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[2] Though we earlier stressed that predicate letters are just 1-place versions of relation letters, in the case of semantics for a predicate letter we leave out the useless 'ordering' found in an 'ordered' 1-tuple such as <**2**>, and populate the predicate letter's extension with the object itself – in this case, **2**.

Likewise in the 3-place relation sentence "__ likes __ more than ___" the same name can fill more than one place. In the following model the sentence "Jake likes Jezebel more than he likes himself" is true.

K^3abc: **a likes b more than c**

Jake likes Jezebel more than he likes himself K^3aba

𝔻: {**JAKE**, **JEZEBEL**}

a: **JAKE** b: **JEZEBEL**
K^3: {**< JAKE**, **JEZEBEL**, **JAKE >**}

Whereas in this model the formal sentence "K^3bab" ("Jezebel likes Jake more than she likes herself") and "K^3abb" ("Jake likes Jezebel more than she likes herself") are **false**.

Jezebel likes Jake more than she likes herself K^3bab
Jake likes Jezebel more than she likes herself K^3abb

Summary: Relation Semantics

Relation letter semantics:

> **The extension (the semantic value) of an *n*-place relation letter in model *M* is a (possibly empty) set of ordered *n*-tuples of objects drawn from the domain of *M*.**

(For example: the extension of a **2-place** relation letter in a certain model is a set of **ordered pairs** of objects from the domain of that model; the extension of a **3-place** relation letter is a set of **ordered triples** of objects from the domain of the model; etc.)

A formal sentence composed of an n-place relation letter, followed by n many name letters, is true in a model just in case the objects named by those name letters appear in an ordered n-tuple of objects (in that order) in the extension of that relation letter.

(For example: the sentence "$\mathbf{G^2ab}$" is true in model M if and only if the extension of "$\mathbf{G^2}$" contains an ordered pair whose first member is the object named "**a**" and whose second member is the object named "**b**".)